Approximate Zero Variance Importance Sampling and Recursive Variance Reduction for diameter constrained reliability estimation

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Abstract—The diameter constrained reliability generalizes the classic network reliability concept admitting constraints in the length of the paths communicating the terminal nodes. In the context of highly reliable networks some traditional techniques do not achieve good results estimating the diameter constrained reliability value. An alternative approach is the use of variance reduction methods which improve the accuracy of Standard Monte Carlo. We present the comparison between two variance reduction methods in a diameter constrained regime, Approximate Zero Variance Importance Sampling and Recursive Variance Reduction. The numerical results presented illustrate the improvement on the global performance of the methods respect to Standard Monte Carlo. The methods are compared in terms of variance and computing time.

Keywords—diameter constrained reliability, Importance Sampling, rare events, Recursive Variance Reduction.

I. INTRODUCTION

Determining the probability of successful communication between a set of nodes is vital in telecommunication network design. The meaning of successful communication depends on the addressed problem, in some cases the condition required is only the exchange of information between nodes while in others low delays are also necessary. This is the case of simultaneous communication by video-conference. The advance in technology has allowed the design of highly reliable networks where the failure of its elements is an unlikely event or as referred in the literature a rare event [1].

The diameter constrained reliability is defined as the probability that two terminal nodes, s and t, of a network G are connected by paths of maximum length D. Although the exact calculation of this measure is polynomial in some networks [2], in general it is complex and the computing time grows exponentially with the number of links which makes it impractical in large graphs. In such cases approximation procedures based on the Standard Monte Carlo method are more suitable. However in highly reliable networks a large number of replications must be considered to achieve an acceptable accuracy level in the estimations. This motivates the use of variance reduction methods among which are Importance Sampling and Recursive Variance Reduction. In order to estimate the diameter constrained reliability measure we implement these techniques and evaluate their performance in accuracy and computing time.

This paper is organized as follows. In Section II we specify the probabilistic model for the diameter constrained reliability. In sections III and IV we define the Approximate Zero Variance Importance Sampling and the Recursive Variance Reduction method. In Section V we report the empirical results of both methods. Conclusions are presented in Section VI.

II. DIAMETER CONSTRAINED RELIABILITY

The diameter constrained reliability, denoted as $R_{(K)}(G,D)$, was introduced by Petingi and Rodríguez in the year 2001 [3], extending the concept of the classical reliability measure. In the mathematical model the network is represented by an undirected graph $G = (V,E)$ with $V = \{v_1,\ldots,v_n\}$ a set of nodes and $E = \{e_1,\ldots,e_m\}$ a set of links. The nodes are assumed perfect and the links fail with a given identical probability. The state of each link is represented by a random variable Bernoulli(r), with r the probability that link $e \in E$ operates and $1-r$ the probability of failure.

Given the set of terminal nodes $K = \{s,t\}$, and the diameter constraint D, the reliability $R_{(s,t)}(G,D)$ is the probability that the set of nodes $\{s,t\}$ of the graph G is connected by paths of maximum length D. The constraint D verifies $D \in Z^+$ and $D \in [1,n-1]$ with $n = |V|$. Additionally the diameter constrained unreliability is defined as $Q_{(K)}(G,D) = 1 - R_{(K)}(G,D)$.

As we mentioned above the exact computation of $R_{(s,t)}(G,D)$ is NP-hard [4]. As an alternative, the Standard Monte Carlo method, also referred as Crude Monte Carlo (MCC), provides an approximation to the value of $R_{(s,t)}(G,D)$ by sampling in the state space of the network. In each replication, the state of every link is sampled independently assuming a Bernoulli(r) distribution and the global state of the network is evaluated. The value of $R_{(s,t)}(G,D)$ is estimated by the proportion of replications corresponding to an operative state of the network. Although simple this method is not always suitable, in highly reliable networks it does not lead to accurate estimations unless a large number of replications is considered.
Related to the reliability measure we consider the cutsets of a network. A D-cutset $C$ is a set of links of $G$ such that its removal results in a failure of the network. If any proper subset of $C$ is not a cutset, then $C$ receives the name of D-mincut.

Since the probability of failure of the links is assumed to be identical and independent, the probability of failure of a cutset $C$, $Q_C$, can be expressed as $(1 - r)^{|C|}$.

III. Approximate Zero Variance Importance Sampling

The Importance Sampling method can achieve an improved precision on the point estimation by translating the problem from a rare event regime to a new one where the failure of the network is more frequent.

The unreliability measure of $G$ can be expressed as $Q_{(s,t)}(G,D) = E(\Phi(X))$, with $X$ a random discrete vector with mass function $P$, and $\Phi(.)$ the structure function with $\Phi(X) = 0$ if $X$ corresponds to an operational state of the network and $\Phi(X) = 1$ otherwise. If $X \in \mathbb{R}^m$ and its elements $X_i$, with $i = 1, \ldots, m$, are i.i.d. with mass function $p$, we have:

$$E(\Phi(X)) = \sum_{x \in \Omega_G} \Phi(x)P(X = x),$$

where $\Omega_G$ is the space of all state vectors of $G$.

Equation 1 can also be written as:

$$E(\Phi(X)) = \sum_{x \in \Omega_G} \Phi(x)P(X = x) \frac{\tilde{P}(X = x)}{P(X = x)} = \sum_{x \in \Omega_G} \Phi(x)L(x)\tilde{P}(X = x),$$

(2)

where $\tilde{P}$ is a new mass function of $X$ and $L(x) = \frac{P(X=x)}{\tilde{P}(X=x)}$ is the likelihood ratio of $P$ and $\tilde{P}$. Under the assumption that $X$ is distributed following a new probability law $\tilde{P}$ we have:

$$Q_{(s,t)}(G,D) = \tilde{E}(\Phi(X)L(X)).$$

(3)

According to Equation 3 an unbiased estimator is defined for $Q_{(s,t)}(G,D)$:

$$\hat{Q}_{(s,t)}(G,D) = \frac{1}{N} \sum_{j=1}^{N} \Phi(x^{(j)})L(x^{(j)}),$$

(4)

where $N$ represents the sample size and $x^{(j)}$ the j-th realization of the state vector of $G$.

The variances corresponding to $P$ and $\tilde{P}$ are respectively:

$$\sigma^2(\Phi(X)) = E[\Phi(X)^2] - (E[\Phi(X)])^2,$$

$$\tilde{\sigma}^2(\Phi(X)L(X)) = \tilde{E}[(\Phi(X)L(X))^2] - (E[\Phi(X)])^2.$$

In fact as $\Phi(x) \geq 0$ then

$$\tilde{E}[(\Phi(X)L(X))^2] = (E[\Phi(X)])^2,$$

which results in $\tilde{\sigma}^2(\Phi(X)L(X)) = 0$.

The change of measure adopted here is based on the formulation presented in [5] for the classical network reliability problem where a sequential sampling strategy of the states of the links is considered under a zero variance scheme. For each iteration $j$ the state of the links is sampled according to a pre-established order and the probability of failure depends on the state of the links already sampled. In the referred formulation the i-th step of the sequence is defined as follows:

$$Q_i(x_1, \ldots, x_{i-1}) = E[\Phi(X)/X_1 = x_1, \ldots, X_{i-1} = x_{i-1}],$$

$$Q_i(x_1, \ldots, x_{i-1}) = q_iQ_{i+1}(x_1, \ldots, x_{i-1}, 0) + (1 - q_i)Q_{i+1}(x_1, \ldots, x_{i-1}, 1)$$

The probability considered to sample the state of link $i$ is defined as $\tilde{p}_i = 1 - q_i$ and:

$$q_i = P[X_i = 0/X_1 = x_1, \ldots, X_{i-1} = x_{i-1}] = \frac{q_iQ_{i+1}(x_1, \ldots, x_{i-1}, 0)}{Q_i(x_1, \ldots, x_{i-1})}$$

(6)

The resulting likelihood ratio of $p$ and $\tilde{p}$ in step $i$ is

$$L_i(x_i) = \frac{x_i - q_i}{1 - q_i} + (1 - x_i)\frac{q_i}{q_i} = \frac{Q_i(x_1, \ldots, x_{i-1})}{Q_{i+1}(x_1, \ldots, x_{i-1}, x_i)}$$

Under this sampling strategy $\Phi(X) = 1$ and $L(X) = Q_{(s,t)}(G,D)$ which implies that $\hat{Q}_{(s,t)}(G,D)$ is a zero variance estimator of $Q_{(s,t)}(G,D)$ [5].

The exact zero variance scheme is impractical to implement due to the new mass function $\tilde{P}$ is written in terms of the parameter $Q_{(s,t)}(G,D)$. An alternative is approximating the value of $Q_i$ in $\tilde{p}$ with the maximum probability of failure of a D-mincut of $G$, assuming fixed the state of the links already sampled.

The problem of identifying a D-mincut of $G$ with maximum probability is solved in polynomial time in the classic reliability model; meanwhile in the diameter constrained model the problem is NP-hard [6]. We propose replacing the value of $Q_{(s,t)}(G,D)$ with an approximation of the maximum probability of a D-mincut conditional to the state of the links already sampled. In order to address this problem we consider an approximation algorithm for the Primal Dual of the Max Flow and Min Cut problem [6].

The resulting estimation method for $Q_{(s,t)}(G,D)$ will be referred as Approximated Zero Variance Importance Sampling (AZVIS). The method is iterative so the computing time is linear in $N$ [5], the states of the links must be sampled and the structure function must be evaluated in each iteration. It also depends on the topology of the graph, larger graphs will involve higher computation time.
IV. RECURSIVE VARIANCE REDUCTION

The Recursive Variance Reduction method (RVR) estimates \( Q_{\{s,t\}}(G, D) \) based on partitions of the state space of the network and the elements of a D-mincut of \( G \). The RVR method implemented is based on the formulation proposed by Cancela and El Khadiri [7] for the classical reliability problem and it considers a sample of \( \Omega_G \) of size \( N \) generated using multinomial sampling.

Given \( P_G \) a partition of \( \Omega_G \) and \( C \) a D-mincut such that \(|P_G| = |C|\) we define:

\[
P_G = \bigcup_{k=1}^{|C|} A_k.
\]

Each element \( A_k \) is a set of subgraphs of \( G \) conditional to the event that the \( k \)-th link of \( C \) is operational and the first \( k-1 \) links fail.

Furthermore the random variables \( N_k \) are defined to represent the number of sampled states belonging to \( A_k \), each \( N_k \) follows a Binomial\((N, p(A_k))\) distribution and verifies:

\[
\sum_{k=1}^{|C|} N_k = N, \quad \sum_{k=1}^{|C|} p(A_k) = 1.
\]

The vector \((N_1, \ldots, N_{|C|})\) follows a Multinomial distribution with parameters \( N \) and \( p = (p(A_1), \ldots, p(A_{|C|})) \).

The recursion is performed on each component of \( P_G \) so that the recursion tree has a number of ramifications equal to the size of \( C \). The process goes on until a deterministic state of the graph is reached. The procedure to identify the D-mincut is the same used in the Approximated Zero Variance Importance Sampling [6]. Let’s define \( Q_{\{s,t\}}(G, D) \), a diameter constrained unreliability estimator [7] as:

\[
N Q_C + (1 - Q_C) \sum_{k=1}^{|C|} S1(G_k, N_k) = \frac{S1}{N},
\]

(7)

where \( Q_C \) is the probability of failure of \( C \), \( S1(G_k, N_k) = \sum_{j=1}^{N_k} Y_{G_k}^{i} \) and \( Y_{G_k}^{j} \) the \( j \)-th realization of the states sampled in \( A_K \). It can be shown that \( N Q_C \) is an unbiased estimator of \( Q_{\{s,t\}}(G, D) \).

The variance estimator proposed in [7] is:

\[
\hat{V}_G = \frac{1}{N(N-1)} \left[ S2 - \left( \frac{S1}{N} \right)^2 \right],
\]

(8)

where

\[
S2(G, N) = N Q_C^2 + 2 Q_C (1 - Q_C) \sum_{i=1}^{|C|} S1(G_i, N_i)
\]

\[
+ (1 - Q_C)^2 \sum_{i=1}^{|C|} S2(G_i, N_i)
\]

The execution time per sample unit of RVR decreases as \( N \) increases. The reason is that the estimations of \( Q_{\{s,t\}}(G, D) \) are obtained without repeating any calculation and when \( N \) is sufficiently large, many replications share the same branch of the recursion tree. [7]

V. NUMERICAL RESULTS

In this section we report the numerical results obtained by implementing the Approximate Zero Variance Importance Sampling and the Recursive Variance Reduction. The selected graphs have been chosen among those more widely used in the literature: Arpanet, Dodecahedron and Grid 5, with different values of diameter constraint and unreliability \( 1 - r \). The algorithms are implemented in MATLAB, and the numerical experiment are computed in the Cluster of Facultad de Ingeniería-UDELAR\(^1\).

In Table I we present results obtained by the AZVIS method with sample size \( 10^2 \). We include the point estimation of \( Q_{\{s,t\}}(G, D) \), the corresponding variance estimation \( \hat{V}_{AZVIS} \), the computing time measured in seconds \( t_{AZVIS} \) and the speed up measure \( W \) estimated by:

\[
\hat{W}_{MCC} = \frac{\hat{V}_{MCC}/V_{MCC}}{\hat{V}_{AZVIS}/V_{AZVIS}},
\]

with \( \hat{V}_{MCC} \) and \( t_{MCC} \) the estimated variance and the computing time of Crude Monte Carlo. An analogous criteria is considered to present the results for RVR in tables II and III with sample sizes \( 10^2 \) and \( 10^4 \) respectively.

The measure \( W \) is used to evaluate the performance of a method respect to MCC considering two factors, computing time and variance. It does not depend on the sample size and it allows the comparison between different methods and number of replications.

Table I: Numerical results for AZVIS with \( N = 10^2 \).

<table>
<thead>
<tr>
<th>Graph</th>
<th>( D )</th>
<th>( 1 - r )</th>
<th>( Q_{AZVIS} )</th>
<th>( V_{AZVIS} )</th>
<th>( t_{AZVIS} )</th>
<th>( W_{MCC} )</th>
</tr>
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<tbody>
<tr>
<td>Arpanet</td>
<td>6</td>
<td>1E-04</td>
<td>5.759E-04</td>
<td>6.237E-09</td>
<td>205</td>
<td>9.15E+03</td>
</tr>
<tr>
<td>Arpanet</td>
<td>6</td>
<td>1E-06</td>
<td>5.199E-04</td>
<td>4.749E-09</td>
<td>175</td>
<td>1.05E+04</td>
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<td>Arpanet</td>
<td>6</td>
<td>1E-08</td>
<td>1.537E-04</td>
<td>4.755E-11</td>
<td>245</td>
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</tr>
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<td>Arpanet</td>
<td>10</td>
<td>1E-04</td>
<td>3.809E-04</td>
<td>6.521E-24</td>
<td>261</td>
<td>1.22E+08</td>
</tr>
<tr>
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<td>1E-04</td>
<td>8.516E-04</td>
<td>6.113E-27</td>
<td>307</td>
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</tr>
<tr>
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<td>1.020E-04</td>
<td>8.885E-39</td>
<td>273</td>
<td>3.53E+16</td>
</tr>
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<td>1E-08</td>
<td>9.703E-04</td>
<td>6.980E-28</td>
<td>203</td>
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</tr>
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<td>Dodec.</td>
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<td>9.500E-04</td>
<td>8.331E-40</td>
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</tr>
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<td>1E-04</td>
<td>9.302E-04</td>
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<td>590</td>
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<td>Grid 5</td>
<td>8</td>
<td>1E-06</td>
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<td>545</td>
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</tr>
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<td>Grid 5</td>
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Table II: Numerical results for RVR with \( N = 10^2 \).

<table>
<thead>
<tr>
<th>Graph</th>
<th>( D )</th>
<th>( 1 - r )</th>
<th>( Q_{RVR} )</th>
<th>( V_{RVR} )</th>
<th>( t_{RVR} )</th>
<th>( W_{MCC} )</th>
</tr>
</thead>
<tbody>
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<td>5.998E-04</td>
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<td>1.000E-04</td>
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<td>1E-06</td>
<td>1.000E-12</td>
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<tr>
<td>Grid 5</td>
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\(^1\) http://www.fing.edu.uy/cluster/index.php
Table III: Numerical Results of RVR with \( N = 10^4 \).

<table>
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<tr>
<th>Graph</th>
<th>( D )</th>
<th>( 1 - r )</th>
<th>( Q_{RVR} )</th>
<th>( V_{RVR} )</th>
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We compare the results obtained implementing RVR with \( N = 10^2 \) and \( N = 10^4 \) on one hand and also the results of RVR run for \( N = 10^4 \) and AZVIS with \( N = 10^2 \).

The results showed in tables II and III reveal that the estimated values of \( W_{MCC} \) for \( N = 10^2 \) are different from the ones corresponding to \( N = 10^4 \). This first approach suggests that \( N = 10^2 \) is not sufficient to stabilize the estimations of RVR, and motivates the use of \( N = 10^4 \). As we mentioned earlier the expected value of \( W \) does not depend on \( N \), and its value should be the same for \( N = 10^4 \) and \( N = 10^6 \).

The comparison between AZVIS with \( N = 10^2 \) and RVR with \( N = 10^4 \) shows that for \( 1 - r = 10^{-6} \) RVR has a better performance, whereas for \( 1 - r = 10^{-4} \) is the opposite. This result suggests that the use of RVR is preferred in highly reliable networks and the use of AZVIS in intermediate cases.

The methods AZVIS and RVR improve the global performance results of MCC in all the experiments considered. In AZVIS, the main contribution is in terms of variance reduction, obtaining accurate results with low sample sizes. However the computing time of AZVIS for a given \( N \) is higher than MCC since a D-mincut must be identified in each iteration. As for the RVR, the improvement is verified in a reduction of computing time and variance. This method requires values of \( N \) superior to those in AZVIS in order to achieve an acceptable accuracy level but the computational cost is significantly lower, indeed the average time obtained with RVR is 140 times lower than with AZVIS.

VI. Conclusion

In this work we compare the performance of two variance reduction methods implemented to estimate the diameter constrained unreliability measure in the context of highly reliable networks. The Importance Sampling formulation selected is based on an approximation of a zero variance scheme according to which the links states are determined sequentially, conditioned on the state of the links already sampled [5]. The RVR algorithm presented consists in a recursive search over a partition of the state space of the graph and the multinomial sampling of the different components of the partition [7].

In both methods the identification of a D-mincut of maximum probability is required. The exact solution of this problem is NP-hard therefore an approximation of polynomial time is considered [6].

Given a certain number of replications, the computing time is lower in RVR than in AZVIS. In contrast AZVIS yields better results in terms of variance. The empirical results reveal that both methods are more efficient than MCC.

We propose two future work directions: experimenting with different algorithms for choosing the mincuts and combining the estimation methods with topology simplification algorithms like irrelevant links elimination adapted to the diameter constrained reliability [8].

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