A note on solving fuzzy differential equations

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Abstract. In this article we present some remark on solving fuzzy differential equations. Particularly, we show that a fuzzy differential equation may not always be replaced by an equivalent system of parametric differential equations. We given a class of fuzzy differential equations where this equivalence is valid and any classical numerical method for differential equations can be adapted to obtain a fuzzy solution.

Keywords: Fuzzy differential equations, solving fuzzy differential equations, extension principle

1 Introduction

We consider the fuzzy differential equation

\[
\begin{cases}
X'(t) = F(t, X) \\
X(0) = X_0,
\end{cases}
\]

(1)

where \( F : [0, T] \times F_C \to F_C \) is a continuous fuzzy function, \( X_0 \in F_C \), \( F_C \) is the space of all fuzzy intervals and \( X' \) denotes the strongly generalized derivative of the fuzzy-valued function \( X \).

It is well-known that the fuzzy differential equation (1) may be replaced by an equivalent system of parametric differential equations [1–3, 6, 7, 11, 12, 15–17, 19, 23–26]. Thus, for we obtain a solution of the fuzzy differential equation (1), when it exists, we have to solve the system of parametric differential equations. This fact has been well used for implement, adapt classical numerical methods for differential equation to fuzzy context, for instance see [1–3, 11, 12, 15–17, 19, 23–26].

However, the fuzzy differential equation (1) may not always be replaced by an equivalent system of parametric differential equations in the form that it is considered in some papers, as we are going to show. But if we consider that \( F \) is generated by applying the Zadeh’s extension principle to a real function of variable real then we have this equivalence.

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2 Preliminares

We denote by $\mathcal{K}_C$ the family of all compact intervals in $\mathbb{R}$, i.e.
$\mathcal{K} = \{ A = [a^-, a^+] \mid a^-, a^+ \in \mathbb{R}, a^- \leq a^+ \};$

On the space $\mathcal{K}_C$, we have the usual interval arithmetic which is due to Moore [21]. He states that, given two intervals $A$ and $B$,

$$A \times B = \{ c \mid c = a \times b, a \in A, b \in B, \times \in \{ +, -, \times, \div \} \},$$

and the multiplication of a real number $\lambda$ by an interval $A$ is given by

$$\lambda A = \{ \lambda a \mid a \in A \} .$$

So we have that the space $(\mathcal{K}_C, +, \cdot)$ is not a linear space.

A fuzzy set $u$ on a universe set $X$ is a mapping $u : X \rightarrow [0, 1]$. We think $u$ as assigning to each element $x \in X$ a degree of membership, $0 \leq u(x) \leq 1$. If $u$ is a fuzzy set on $\mathbb{R}$, we define $[u]^\alpha = \{ x \in \mathbb{R} \mid u(x) \geq \alpha \}$ the $\alpha$-level of $u$, with $0 < \alpha \leq 1$. For $\alpha = 0$ we have the support of $u$ is defined as $[u]^0 = \text{supp}(u) = \{ x \in \mathbb{R} \mid u(x) > 0 \}$, where $\overline{A}$ denotes the closure of $A \subset \mathbb{R}$.

A fuzzy interval $u$ is a fuzzy set on $\mathbb{R}$ such that $[u]^\alpha \in \mathcal{K}_C$ for all $\alpha \in [0, 1]$. We denote by $\mathcal{F}_C$ the family of all fuzzy intervals, i.e.

$\mathcal{F}_C = \{ u : \mathbb{R} \rightarrow [0, 1] \mid [u]^\alpha \in \mathcal{K}_C, \forall \alpha \in [0, 1] \}.$

We denote by $[u]^\alpha = [u_-, u_+]$ the $\alpha$-level of a fuzzy interval $u$. If the core of a fuzzy interval $u$ is a unitary set, i.e. if $[u]^1 = \{ a \}$, with $a \in \mathbb{R}$, then $u$ is called fuzzy number. An special fuzzy number is the triangular fuzzy number which is well-defined by three real numbers. So we denote a triangular fuzzy number by $u = (a, b, c)$ with $a, b, c \in \mathbb{R}$.

Given two fuzzy intervals $u$ and $v$ we can define the pair $(u, v)$, a fuzzy set on $\mathbb{R}^2$, such that

$$(u, v)(x, y) = u(x) \land v(y),$$

where $\land$ denotes the minimum. Then $[(u, v)]^\alpha = [u]^\alpha \times [v]^\alpha$, for all $\alpha \in [0, 1]$, where $\times$ denotes the usual Cartesian product [10]. So, we can define a fuzzy vector as being $(u_1, ..., u_n)$ such that $u_i \in \mathcal{F}_C$, for $i = 1, ..., n$. We will denote by $(\mathcal{F}_C)^n$ the space of all fuzzy vector, i.e.

$$(\mathcal{F}_C)^n = \{ (u_1, ..., u_n) \mid u_i \in \mathcal{F}_C, \ i = 1, ..., n \}.$$

2.1 Zadeh’s extension principle

In [28], Zadeh proposed an extension principle, which has become an important tool in fuzzy theory and its applications. The idea is that each function $f : \mathbb{R}^m \rightarrow \mathbb{R}^n$ induces a unique fuzzy function $\hat{f} : (\mathcal{F}_C)^m \rightarrow (\mathcal{F}_C)^n$, defined for each fuzzy vector $(u_1, ..., u_m)$ by

$$\hat{f}(u_1, ..., u_n)(y) = \begin{cases} \sup_{x \in \mathbb{R}^m, f(x) = y} (u_1, ..., u_n)(x) & \text{if } f^{-1}(y) \neq \emptyset \\ 0 & \text{if } f^{-1}(y) = \emptyset. \end{cases}$$

(4)
We said that the fuzzy function \( \hat{f} \) is obtained from \( f \) by applying the Zadeh’s extension principle.

Since arithmetic operations are continuous real-valued functions, excluding division by zero, the extension principle can be used for obtain a fuzzy arithmetic [10, 13, 18]. For instance, if we consider the function \( f_+ : \mathbb{R}^2 \to \mathbb{R} \) defined by \( f_+(x, y) = x + y \), applying the Zadeh’s extension principle we obtain a unique fuzzy function \( \hat{f}_+ : (\mathcal{F}_C)^2 \to \mathcal{F}_C \) defined by
\[
\hat{f}_+(u, v)(x) = \sup_{x_1 + x_2 = x} u(x_1) \land v(x_2).
\]

So, we define the sum between two fuzzy interval \( u, v \in \mathcal{F}_C \) by
\[
\hat{f}(u \oplus v) = \hat{f}_+(u, v).
\]

Also, considering the function \( f_m \) defined by \( f_m(x) = \lambda \cdot x \), with \( \lambda \in \mathbb{R} \), and applying the Zadeh’s extension principle we obtain \( \lambda \circ u = \hat{f}_m(u) \), the multiplication of a scalar \( \lambda \) by a fuzzy interval \( u \).

In particular any continuous function \( f \) can be extended to a unique fuzzy function. For example, if we consider the function \( f_1 : \mathbb{R}^+ \to \mathbb{R} \) defined by \( f_1(x) = \sqrt{x} \), applying the Zadeh’s extension principle we obtain the fuzzy function \( \hat{f}_1 : \mathcal{F}_C(\mathbb{R}^+) \to \mathcal{F}_C \) and we define \( \sqrt{u} \) by
\[
\sqrt{u} = \hat{f}_1(u),
\]
for all fuzzy interval \( u \) such that \([u]^0 \subset \mathbb{R}^+\).

In general, the calculus of the fuzzy function \( \hat{f} \) is a complex problem [9, 13, 14]. But there exists a relation between the \( \alpha \)-levels of \( \hat{f}(u) \) and the image of \( \alpha \)-levels of \( u \) by \( f \), which help to obtain \( \hat{f} \).

\textbf{Theorem 1.} ([27]) \( f : \mathbb{R}^n \to \mathbb{R}^m \) is a continuous function if and only if \( \hat{f} : (\mathcal{F}_C)^n \to (\mathcal{F}_C)^m \) is a well-defined function and it is continuous in relation to \( D \)-metric. Moreover
\[
\left[ \hat{f}(u_1, ..., u_n) \right]^\alpha = f \left( [u_1]^\alpha, ..., [u_n]^\alpha \right), \quad \forall \alpha \in [0, 1], \forall u_i \in \mathcal{F}_C,
\]
where \( f(A) = \{ f(a) / a \in A \} \).

From equation (7) we obtain that the sum of two fuzzy intervals \( u \oplus v \), obtained applying the Zadeh’s extension principle (5), verifies the following property (see [10, 13])
\[
[u \oplus v]^\alpha = [u]^\alpha + [v]^\alpha, \quad \forall \alpha \in [0, 1],
\]
where \([u]^\alpha + [v]^\alpha\) is the sum of two compact intervals defined by (2). Also, the multiplication of a scalar by a fuzzy set \( \lambda \circ u \) has the following property (see [10, 13]):
\[
[\lambda \circ u]^\alpha = \lambda [u]^\alpha, \quad \forall \alpha \in [0, 1],
\]
where \( \lambda [u]^\alpha \) is the multiplication of a scalar by a compact interval (3). We can see that \((\mathcal{F}_C, \oplus, \circ)\) is not a linear space [10].

On the fuzzy interval space \( \mathcal{F}_C \), additional to sum and multiplication by a scalar, applying the Zadeh’s extension principle we can define the subtraction
Remark 1. When we write \( u \otimes (v \oplus w) \), mean that it was obtained applying the Zadeh’s extension principle to the function \( f(x, y, z) = x(y + z) \) and so \( u \otimes (v \oplus w) = \hat{f}(u, v, w) \). Since \( f(x, y, z) = x(y + z) = xy + xz \), then from (4) we have
\[
 u \otimes (v \oplus w) = u \otimes v \oplus u \otimes w,
\]
for all \( u, v, w \in F_C \).

Also, if in the fuzzy algebraic expression a unique fuzzy interval is involved then it was obtained from a real function of variable real. For example, \( u^3 \ominus u^2 \) is obtained applying the Zadeh’s extension principle to function \( f(x) = x^3 - x^2 \). So, from (7), we have
\[
 u^3 \ominus u^2 = u \ominus (u^2 \ominus u) = u^2 \ominus (u \ominus 1),
\]
for all \( u \in F_C \).

We can define other fuzzy arithmetic on the fuzzy interval space \( F_C \): fuzzy interval arithmetic [10, 18]. Given two fuzzy intervals \( u, v \in F_C \) we define \( u \ast v \), with \( \ast \in \{+,-,\times,\div\} \), via its \( \alpha \)-levels by
\[
[u \ast v]^\alpha = [u]^\alpha \ast [v]^\alpha, \quad \text{for all } \alpha \in [0,1],
\]
where \( [u]^\alpha \ast [v]^\alpha \) is obtained from usual interval arithmetic (2).

Note that the sum of two fuzzy intervals \( u \) and \( v \) using Zadeh’s extension principle and using fuzzy interval arithmetic are equivalent, i.e \( u \oplus v = u + v \). In case of the multiplication of a scalar by a fuzzy interval also are equivalent. But, in case of the multiplication of two fuzzy intervals both, using Zadeh’s extension principle and using fuzzy interval arithmetic are not equivalent. For example, if \( u = (-1,0,1) \) then \( u \times u = (-1,0,1) \) while \( u \otimes u = (0,0,1) \). On the other hand, when we combined these algebraic operations, these also are not equivalent. For instance, from Remark 1, \( u \otimes (v \oplus w) = u \otimes v \oplus u \otimes w \), while \( u \times (v + w) \neq u \times v + u \times w \).

In general the fuzzy interval arithmetic yield to an overestimation in relation to the arithmetic generate by using the Zadeh’s extension principle. For more details see [9, 13, 14, 18].

3 Fuzzy differential equations

Let \( u, v \in F_C \) if there exist \( w \in F_C \) such that \( u = v + w \) then we say that there exists the H-difference between \( u \) and \( v \) which we denote by \( u -_H v = w \).

**Definition 1.** [5] Let \((\tau_1, \tau_2)\) be an open interval and let \( t_0 \in (\tau_1, \tau_2) \). Given \( X : (\tau_1, \tau_2) \rightarrow F_C \) a fuzzy-valued function, we say that \( X \) is strongly generalized differentiable (G-differentiable) at \( t_0 \) if exists an element \( X'(t_0) \in F_C \) such that:
(i) for all $h > 0$ sufficiently small, $\exists X(t_0 + h) - H X(t_0), X(t_0) - H X(t_0 - h)$ and the limits (in the metric $D$)

$$\lim_{h \to 0^+} \frac{X(t_0 + h) - H X(t_0)}{h} = \lim_{h \to 0^+} \frac{X(t_0) - H X(t_0 - h)}{h} = X'(t_0)$$

or

(ii) for all $h > 0$ sufficiently small, $\exists X(t_0 + h) - H X(t_0), X(t_0) - H X(t_0 - h)$ and the limits (in the metric $D$)

$$\lim_{h \to 0^+} \frac{X(t_0) - H X(t_0 + h)}{-h} = \lim_{h \to 0^+} \frac{X(t_0) - H X(t_0 - h)}{-h} = X'(t_0).$$

or

(iii) for all $h > 0$ sufficiently small, $\exists X(t_0 + h) - H X(t_0), X(t_0) - H X(t_0 - h)$ and the limits (in the metric $D$)

$$\lim_{h \to 0^+} \frac{X(t_0 + h) - H X(t_0)}{h} = \lim_{h \to 0^+} \frac{X(t_0) - H X(t_0 - h)}{-h} = X'(t_0),$$

or

(iv) for all $h > 0$ sufficiently small, $\exists X(t_0 + h) - H X(t_0), X(t_0) - H X(t_0 - h)$ and the limits (in the metric $D$)

$$\lim_{h \to 0^+} \frac{X(t_0) - H X(t_0 + h)}{-h} = \lim_{h \to 0^+} \frac{X(t_0) - H X(t_0 - h)}{h} = X'(t_0).$$

We say that $X$ is $G$-differentiable on $(\tau_1, \tau_2)$ if $X$ is $G$-differentiable at each point $t_0 \in (\tau_1, \tau_2)$.

Note that the $G$-differentiability in the first form (i) of the Definition 1 is coincident with the Hukuhara differentiability ($H$-differentiability, for short). Thus, the $G$-differentiability is a concept of differentiability for fuzzy-valued functions more general than the $H$-differentiability.

**Definition 2.** We said that a fuzzy function $X : [0, T] \to \mathcal{F}_C$ is a fuzzy solution of the fuzzy differential equation (1) if it is $G$-differentiable on $[0, T]$ and verifies the equation (1) for all $t \in [0, T]$.

Next we describe the procedure for solving the fuzzy differential equation (1) which have been well considered for many authors.

For this, we denote by

$$[X(t)]^\alpha = [x^-_\alpha(t), x^+_\alpha(t)],$$

the $\alpha$-levels of the fuzzy-valued function $X : [0, T] \to \mathcal{F}_C$.

**Theorem 2.** ([7]) Let $X : T \to \mathcal{F}_C$ be a fuzzy function. Then

(i) If $X$ is differentiable in the first form (i), then $x^-_\alpha$ and $x^+_\alpha$ are differentiable functions and

$$\left[X'(t)\right]^\alpha \equiv \left[(x^-_\alpha)'(t), (x^+_\alpha)'(t)\right].$$

(9)
(ii) If $X$ is differentiable in the second form (ii), then $x^-_\alpha$ and $x^+_\alpha$ are differentiable functions and

$$[x'(t)]^\alpha = [(x^-_\alpha)'(t), (x^+_\alpha)'(t)].$$

(10)

In many articles (see for instance [1, 2, 3, 6, 7, 13, 14, 17, 18, 19, 21, 25, 26, 27, 28]) was considered (denoted) the following relation:

$$[F(t, X(t))]^\alpha = [f^-_\alpha(t, x^-_\alpha(t), x^+_\alpha(t)), f^+_\alpha(t, x^-_\alpha(t), x^+_\alpha(t))],$$

and $[X_0]^\alpha = [x^-_0, x^+_0]$.

If there exists a fuzzy solution $X$ of (1), then (1) is equivalent to the following system of differential equations:

(1) is equivalent to the following system of differential equations

$$(Case I)$$ If we consider $X'(t)$ the derivative in the first form (i) (Definition 1), then from (9) we have $[X'(t)]^\alpha = [(x^-_\alpha)'(t), (x^+_\alpha)'(t)]$. Thus, the problem (1) is equivalent to the system of differential equations

$$\begin{cases}
(x^-_\alpha)'(t) = f^-_\alpha(t, x^-_\alpha(t), x^+_\alpha(t)), & x^-_\alpha(0) = x^-_0 \\
(x^+_\alpha)'(t) = f^+_\alpha(t, x^-_\alpha(t), x^+_\alpha(t)), & x^+_\alpha(0) = x^+_0
\end{cases}$$

(12)

where $x^-_\alpha$ and $x^+_\alpha$ are variables for each $\alpha \in [0, 1]$.

(1) is equivalent to the following system of differential equations

$$(Case II)$$ If we consider $X'(t)$ the derivative in the second form (ii) (Definition 1), then from (10) we have $[X'(t)]^\alpha = [(x^-_\alpha)'(t), (x^+_\alpha)'(t)]$. Thus, the problem (1) is equivalent to the system of differential equations

$$\begin{cases}
(x^-_\alpha)'(t) = f^+_\alpha(t, x^-_\alpha(t), x^+_\alpha(t)), & x^-_\alpha(0) = x^-_0 \\
(x^+_\alpha)'(t) = f^-_\alpha(t, x^-_\alpha(t), x^+_\alpha(t)), & x^+_\alpha(0) = x^+_0
\end{cases}$$

(13)

where $x^-_\alpha$ and $x^+_\alpha$ are variables for each $\alpha \in [0, 1]$.

Note that, for each $\alpha \in [0, 1]$, the right-hand of the system (12) and (13) dependent only on the variables $x^-_\alpha$ and $x^+_\alpha$. Therefore, for each $\alpha \in [0, 1]$ fixed, (12) and (13) are a system of differential equations on two variables, $x^-_\alpha$ and $x^+_\alpha$. Thus, any classical numerical method can be applied for obtain a solution of each system (12) and (13) for each $\alpha \in [0, 1]$ fixed. Consequently, we obtain one or two fuzzy solutions of (1), when this there exist.

Using the procedures previously described, Case I and Case II, many paper have been dedicated to adapt classical numerical methods for differential equations to fuzzy context. However, the fuzzy differential equation (1) is not always equivalent to the system of parametric differential equations (12) or (13), as we can see in the following examples.

**Example 1.** We consider the fuzzy function $F : \mathcal{F}_C \to \mathcal{F}_C$ defined via its $\alpha$-levels by

$$[F(X)]^\alpha = [x^-_1, x^+_1],$$

(14)

that is, $F$ associate to each fuzzy interval $X$ the side right of it. For instance, if we take the fuzzy interval $X$ such that $[X]^\alpha = [x^-_\alpha, x^+_\alpha] = [\alpha - 1, 2 - \alpha]$ then we have

$$[F(X)]^\alpha = [0, 2 - \alpha].$$
Clearly, $F$ is continuous and Lipschitz. Therefore, the fuzzy differential equation

$$X'(t) = F(X), \quad X(0) = (-1, 0, 1), \quad (15)$$

have a unique solution when we consider the derivative in the first form (i) ($H$-derivative). Associate to (15) we have the following system of differential equations

$$\begin{cases}
(x^-_\alpha)'(t) = x^-_1(t), \quad x^-_\alpha(0) = \alpha - 1 \\
(x^+_\alpha)'(t) = x^+_\alpha(t), \quad x^+_\alpha(0) = 1 - \alpha.
\end{cases} \quad (16)$$

Here $x^-_\alpha$ and $x^+_\alpha$ are variables for each $\alpha \in [0, 1]$.

Note that the hand-right of the family of differential equations

$$\begin{cases}
(x^-_\alpha)'(t) = x^-_1(t), \quad x^-_\alpha(0) = \alpha - 1,
\end{cases} \quad (17)$$

it does not depend on $x^-_\alpha$ or $x^+_\alpha$, it dependent only on the variable $x^-_1$, for each $\alpha \in [0, 1]$. For example, if we consider $\alpha = 1/2$, then we have the differential equation

$$(x^-_{1/2})'(t) = x^-_1(t), \quad x^-_{1/2}(0) = -1/2,$$

and if we take $\alpha = 1/3$, then we have the differential equation

$$(x^-_{1/3})'(t) = x^-_1(t), \quad x^-_{1/3}(0) = -2/3.$$

Then the system of fuzzy differential equations (16) is not equivalent to the system of fuzzy differential equations (12) since that the hand-right of (16) dependent on the variable $x^-_1$ for each $\alpha \in [0, 1]$ while the hand-right of (12) dependent only on $x^-_\alpha$ and $x^+_\alpha$ for each $\alpha \in [0, 1]$.

In this particular case, we can solve the system of differential equations (16). In fact, for $\alpha = 1$ we have the system

$$\begin{cases}
(x^-_1)'(t) = x^-_1(t), \quad x^-_1(0) = 0 \\
(x^+_1)'(t) = x^+_1(t), \quad x^+_1(0) = 0,
\end{cases}$$

from where we obtain $(x^-_1)'(t) = 0$ and $(x^+_1)'(t) = 0$. Then, for $0 \leq \alpha < 1$ we have the system

$$\begin{cases}
(x^-_\alpha)'(t) = 0, \quad x^-_\alpha(0) = \alpha - 1 \\
(x^+_\alpha)'(t) = x^+_\alpha(t), \quad x^+_\alpha(0) = 1 - \alpha,
\end{cases}$$

from where we obtain $(x^-_\alpha)'(t) = \alpha - 1$ and $(x^+_\alpha)'(t) = (1 - \alpha)e^t$. Therefore the fuzzy solution $X$ of the fuzzy differential equation (15) is given by $X(t) = (-1, 0, e^t)$, for all $t \geq 0$.

From Example 1 we can see that the error is in the assumption (11). The functions $f^-_\alpha$ and $f^+_\alpha$ should dependent on all variables $x^-_\alpha$ and $x^+_\alpha$.

There exists a class of fuzzy differential equations where the procedures Case I and Case II are valid, more precisely, where the equation (11) is valid. This class
consists of replacing the fuzzy function \( F \) in (1) by a fuzzy function generated from a real function by applying the Zadeh’s extension principle, that is:

\[
\begin{aligned}
X'(t) &= \hat{f}(t, X) \\
X(0) &= X_0,
\end{aligned}
\]

where \( X_0 \in \mathcal{F}_C \) and \( \hat{f} : [0, T] \times \mathcal{F}_C \to \mathcal{F}_C \) is a continuous fuzzy function obtained by applying the Zadeh’s extension principle to a function \( f : [0, T] \times \mathbb{R} \to \mathbb{R} \).

From (7) we have

\[
[\hat{f}(t, X)]^\alpha = f(t, [X]^\alpha) = f(t, [x^-_\alpha, x^+_\alpha]) = [\min f(t, [x^-_\alpha, x^-_\alpha]), \max f(t, [x^-_\alpha, x^-_\alpha])].
\]

Then, the equation (11) is valid for \( F = \hat{f} \). Consequently, the Case I and Case II are procedures for solving the fuzzy differential equation (18).

Note also that if \( F \) is obtained from fuzzy interval arithmetic, then \( F \) verifies the equation (11). However, the problem (18) is more general than consider \( F \) being obtained from fuzzy interval arithmetic.

The problem (18) is not novel, it has been considered in many papers, see for example [8, 20, 24, 26]. Particularly, in [24, 26] have been implement numerical methods for solving the fuzzy differential equation (18).

4 Final comments

As we have saw, the assumption (11), in general, is not valid. There exits in the literature some paper where the authors considered that (11) is valid for any fuzzy function \( F \) [1–3, 11, 12, 17, 19, 25]. Also, in [1–3, 11], the authors assume that \( F \) can be approximated by a fuzzy function \( G \) such that \( G(X) \) is a pyramidal fuzzy set and then \( G \) satisfies (11) [4]. However the fuzzy function (14), given in the Example 1 is a fuzzy function such that \( F(X) \) is a pyramidal fuzzy set and these do not satisfies (11). Moreover, there exist other authors that included clearly in its papers the equation (11) as a condition for implement a numerical method [23, 24, 26].

On the other hand, when the equation (11) is valid, we can apply any classical numerical method for solve the systems (12) and (13) since, for each \( \alpha \in [0, 1] \) fixed, (12) and (13) are a system of differential equations in two variables. So, adapt any classical numerical method for solving the fuzzy differential equation (1) such that (11) is verify should not be a complex problem.

References

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